His own musical interests include playing clarinet, saxophone, and recorders, and singing. He has directed church choirs, plays in the Kishwaukee Symphony Orchestra, and frequently appears as tenor soloist in oratorios. Another musical interest is bell ringing. He learned change ringing on the historic bells of Old North Church in Boston, and he frequently rings handbells.

[1]J. Backus, *The Acoustical Foundations of Music,* 2d ed. (New York: W. W. Norton and Co., 1977); A. H. Benade, *Fundamentals of Musical Acoustics* (New York: Oxford University Press, 1976); J. Roederer, *Introduction to the Physics and Psychophysics of Music,* 2d ed. (New York: Springer Verlag, 1975); W. J. Strong and G. R. Plitnik, *Music, Speech, and High Fidelity* (Provo, Utah: Brigham Young University Press, 1977); D. E. Hall, *Musical Acoustics: An Introduction* (Belmont, Calif.: Belmont, 1980); H. E. White and D. H. White, *Physics and Music* (Philadelphia: Saunders, 1980); J. S. Rigden, *Physics and the Sound of Music* (New York: Wiley, 1977); N. H. Fletcher, *Physics and Music* (Richmond, Vic.: Heinemann Educational Australia, 1976); and T. D. Rossing, *The Science of Sound* (Reading, Mass.: Addison-Wesley, 1982). For a list of textbooks published before 1975 see T. D. Rossing, "Resource Letter MA-1: Musical Acoustics," *American Journal of Physics* 43 (1975):944-53. Also included is a bibliography of over 200 selected references on musical acoustics.

[2]Rossing, *Science of Sound.*

[3]Idem, "Acoustics of Percussion Instruments," *Instrumentalist* 30 (May 1976):55-59 and (June 1976):62-65.

[4]Idem, "Acoustics of Percussion Instruments," *Physics Teacher* 14 (1976):546-56 and 15 (1977):278-88.

ACOUSTICS OF BAR PERCUSSION INSTRUMENTS

Thomas D. Rossing

Introduction

Musical instruments generally use one or more of the following basic types of vibrators: strings, bars, membranes, plates, air columns, or air chambers. Two of them (strings and air columns) tend to produce harmonic overtones; the others, in general, do not. Bars, membranes, and plates are three classes of vibrators whose modes of vibration are not related harmonically; thus the overtones they sound will not be harmonics of the fundamental tone. The inharmonic overtones of these complex vibrators give percussion instruments their distinctive timbres.[1]

This paper, the first of a series on the acoustics of percussion instruments, discusses the modes of vibration of bars, giving simple formulas for the vibrational frequencies in simple prismatic bars and describing how bars are tuned by undercutting. The functions of resonators are also discussed. Instruments discussed include glockenspiels, xylophones, marimbas, vibes, triangles, and chimes.

Vibrations of Bars

A bar (or rod or tube) can vibrate either longitudinally (by expanding and contracting in length) or transversely (by bending at right angles to its length). In percussion instruments, the transverse modes of vibration are nearly always preferred (one exception being the aluminum stroke rods, which are excited longitudinally by stroking with a rosined cloth or gloves). Longitudinal vibrations are much higher in frequency than transverse vibrations, and the various

longitudinal modes are related harmonically. The frequency of vibration depends on the length of the rod and on the elasticity of the material from which it is fabricated but is independent of the thickness, surprisingly enough. In fact, the frequency of vibration of a longitudinal rod is expressed by the single formula $f_n = nv/2L$, where v is the speed of sound in the rod, L is its length and $n = 1, 2, 3, \ldots$, denotes the number of the harmonic (beginning with $n = 1$ for the fundamental frequency). The speed of sound is given by: $v = \sqrt{Y/\rho}$, where Y is Young's elastic modulus and ρ is the density.[2]

When a bar vibrates longitudinally, its motion is almost identical to the movement of the air in a pipe that is open at both ends (a flute, for example). Maximum movement occurs at the ends of the bar or air column (called antinodes), and one or more points in between have a minimum of movement (called nodes). For the fundamental mode, there is one node, and that occurs at the center. For the first overtone (harmonic number 2), there are two nodes with another antinode at the center. One can selectively excite any desired mode of vibration in a bar by clamping it at the location of one of the nodes for that particular mode. If the fundamental mode is desired, the bar should be supported at the center only; this is the way stroke rods are generally mounted.

Transverse vibrations in a bar are a little more complicated. First of all, there are three possible end conditions for the bar: clamped, simply supported, and free. However, nearly all percussion instruments use bars with free ends, so only the free bar will be considered. (One exception is the electronic carillon, which sometimes uses bars or rods that are free at one end and clamped at the other.)

The frequency of a bar in either longitudinal or transverse vibration depends on its length and the density and elasticity of the material, but in transverse vibration, the frequency depends on the thickness of the bar as well. The frequency of transverse vibrations in a bar with free ends is given by the formula $f_n = (\pi v K/8L^2)m^2$, where v is the speed of sound, L is the length of the bar, m is a sequence number to be defined in a moment, and K is the radius of gyration, which is related to the size and shape of the bar.[1] (For a flat bar, K equals the thickness divided by 3.46; values for other shapes are given in table 1.) Note the following differences from the longitudinally vibrating bar:

1. The frequency depends on L^2 rather than L.
2. The mode frequencies are not harmonic, but increase as m^2.
3. The frequency depends on the shape of the bar through the factor K.

The frequencies of the modes are almost in proportion to the squares of the odd integers. The number m begins with 3.0112 and then continues with the simple values 5, 7, 9, . . . , $(2n+1)$. Transverse vibrations of a bar are illustrated in figure 1.

The frequencies of the transverse modes of vibration of a uniform bar have the ratios 1.0 : 2.76 : 5.40 : 8.93, and so on, which are anything but harmonic and, in fact, match no intervals on the musical scale. They give a distinctive timbre to instruments such as chimes, orchestra bells and triangles, which use nearly uniform bars. (The bars of marimbas, xylophones, and related instruments are not uniform; they have been cut to have quite a different set of mode frequency ratios and thus a different timbre, as will be discussed.)

← *String*

Transverse vibration: $f_n = n\frac{v}{2L}$

v = speed of waves in the string = $\sqrt{T/\mu}$
L = length of the string
n = 1, 2, 3, . . .
T = tension
μ = mass per unit length

Longitudinal vibration: $f_n = n\frac{v_L}{2L}$

v_L = speed of sound in the string = $\sqrt{Y/\rho}$
(does not change with tension)
Y = Young's modulus of elasticity
ρ = density

Bar with free ends

Transverse vibration: $f_n = \frac{\pi v K}{8L^2} m^2$

v = speed of sound = $\sqrt{Y/\rho}$
Y = Young's modulus of elasticity
ρ = density
L = length
m = 3.0112, 5, 7, . . . , $(2n + 1)$
K = radius of gyration
$K = t/\sqrt{12}$ for a rectangular bar
t = thickness
$K = \frac{1}{2}\sqrt{a^2 + b^2}$ for a tube
a = inner radius
b = outer radius

Longitudinal vibration: $f_n = n\frac{v}{2L}$

v = speed of sound = $\sqrt{Y/\rho}$
L = length

TABLE 1
FORMULAS FOR VIBRATION FREQUENCIES OF STRINGS AND BARS

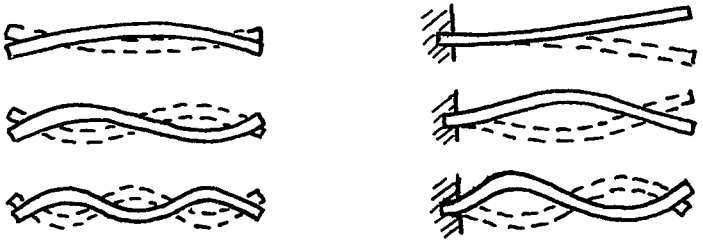


Fig. 1. Modes of vibrating bars: (a) both ends free and (b) one end clamped.

Longitudinal and transverse vibrations may be strikingly demonstrated with aluminum rods about 1 cm in diameter. Longitudinal vibrations can be excited by stroking the rod with a rosined cloth or tapping lightly on the end with a hammer; transverse vibrations, which in a long rod occur at a much lower frequency, can be excited by hitting the rod with one's hand near the center. Since the frequency of transverse vibrations various with $1/L^2$ but the frequency of longitudinal vibrations only varies with $1/L$ (see table 1), it is possible to cut a rod to a length such that the two fundamental frequencies coincide. Note that the nodes for longitudinal and transverse vibration occur at different places, so one's grip must be changed.

Mechanical properties of several materials commonly used in percussion instruments are given in table 2. Somewhat surprising is the relatively small variation in the speed of sound. Bars of aluminum, steel, glass, and redwood, for example, would sound within a quarter of a semitone of each other if they had the same thickness and length. Incorrect statements in several widely used textbooks would lead one to believe otherwise, however.

Rectangular Bars: The Glockenspiel

The *glockenspiel,* or orchestra bells, uses rectangular steel bars 2.5 to 3.2 cm. (1 to 1¼ in.) wide and 0.61 to 1 cm. (¼ to ⅜ in.) thick. Its range is customarily from G_5 (f = 784 Hz) to C_8 (f = 4186 Hz), although it is scored two octaves lower than it sounds. The glockenspiel is usually played with brass or hard plastic mallets. The bell lyra is a portable version that uses aluminum

TABLE 2
PROPERTIES OF MATERIALS

Material	Density (kg/m³)	Young's modulus Y (N/m²)		Sound velocity v (m/s)	
Aluminum [a]	2700	7.1×10^{10}		5150	
Brass [a]	8500	10.4×10^{10}		3500	
Copper [a]	8900	12.2×10^{10}		3700	
Steel [a]	7700	19.5×10^{10}		5050	
Glass [a]	2300	6.2×10^{10}		5200	
Wood:		II grain	I grain	II grain	I grain
Brazilian rosewood [b]	830	1.6×10^{10}	2.8×10^{9}	4400	1800
Indian rosewood [b]	740	1.2×10^{10}	1.7×10^{9}	4000	1500
African mahogany [b]	550	1.2×10^{10}	1.2×10^{9}	5000	1500
European maple [b]	640	1.0×10^{10}	2.2×10^{9}	4000	1800
Redwood [b]	380	0.95×10^{10}	0.96×10^{9}	5000	1600
Sitka spruce [b]	470	1.3×10^{10}	1.3×10^{9}	5200	1700

a L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics,* 2d ed. (New York: Wiley, 1962), p. 502.

b D. W. Haines, "On Musical Instrument Wood," *Catgut Acoustical Society Newsletter,* no. 31 (1979), p. 23.

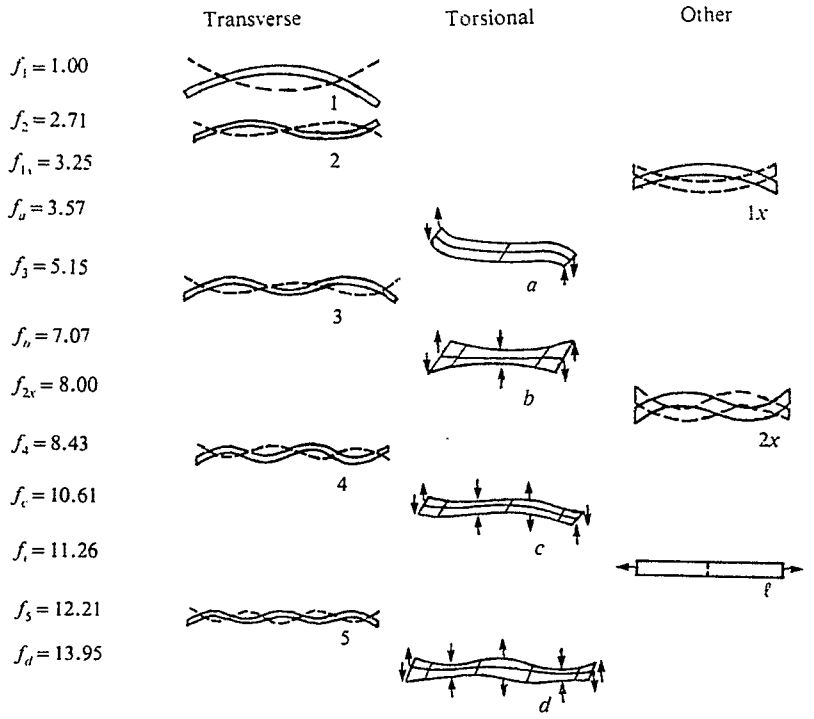
bars, usually covering the range A_5 (f = 880 Hz), to A_7 (f = 3520 Hz), and is used in marching bands.

When struck with a hard mallet, a glockenspiel bar produces a crisp, metallic sound, which quickly gives way to a clear ring at the designated pitch. Because the overtones have very high frequencies and die out quickly, they are of relatively less importance in determining the timbre of the glockenspiel than are the overtones of the marimba or xylophone, for example. For this reason, no effort is made to bring the inharmonic overtones of a glockenspiel into a harmonic relationship through overtone tuning.

The frequencies for transverse vibrations in a bar with free ends were shown to be inharmonic; thus the glockenspiel has no harmonic overtones. However, even its lowest note has a frequency of 2160 Hz, so its overtones occur in a range in which the pitch discrimination of human listeners is diminished. (This will not be the case for other bar percussion instruments, however, as we will see.)

The vibrational modes of a glockenspiel bar are shown in table 3. These modes can be excited individually by passing an electric current from an audio amplifier through the bar while it is in the field of a magnet. Besides the transverse modes, which are labeled 1,2,3,4 and 5, there are torsional or "twisting" modes labeled a,b,c,d; a longitudinal mode f; and transverse modes in the plane of the bar, 1x and 2x.

TABLE 3
VIBRATIONAL MODES OF A GLOCKENSPIEL BAR



The transverse modes of vibration of the glockenspiel bar are spaced a little closer together than predicted by the simple theory of thin bars. The more sophisticated theory for thick bars (called Timoshenko bars in engineering textbooks) takes into account other factors such as the moment of inertia of the bar and the shear stresses in the bar.

Marimba

The *marimba* typically includes 3 to 4⅓ octaves of tuned bars of rosewood or fiberglass synthetic (typical tradenames: Kelon, Klyperon), graduated in width from about 4.5 to 6.4 cm (1¾ to 2½ in.). Beneath each bar is a tubular resonator tuned to the fundamental frequency of that bar. When the marimba is played with soft mallets, it produces a rich mellow tone. The playing range of a large concert marimba is A_2 to C_7 (f = 110 to 2093 Hz), although bass marimbas extend down to C_2 (f = 65 Hz).

A deep arch is cut in the underside of marimba bars, particularly in the low register. This arch serves two useful purposes: It reduces the length of bar required to reach the low pitches, and it allows tuning of the overtones (the first overtone is nominally tuned two octaves above the fundamental). Figure 2 shows a scale drawing of a marimba bar, and also indicates the positions of the nodes for each of the first seven modes of vibration. (These can be compared with the uniform bar shown in figure 1a.) The ratios of the frequencies of these modes are also indicated. Note that the second partial (first overtone) of this bar has a frequency 3.9 times that of the fundamental, which is close to a two-octave interval (a ratio of 4.0).

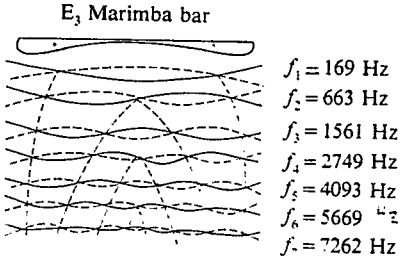


Fig. 2. Scale drawing of a marimba bar tuned to E_3 (f = 265 Hz.) The dashed lines locate the nodes of the first seven notes.[4]

The sound spectrum of the E_3 marimba bar, shown in figure 3 along with the partials on musical staffs, indicates the presence of a strong third partial, which has a frequency of 9.2 times the fundamental (about three octaves plus a minor third above it). The relative strengths of the partials, of course, depend on where the bar is struck and what type of mallet is used. To emphasize a particular partial, the bar should be struck at a point of maximum amplitude for that mode, as indicated in figure 2.

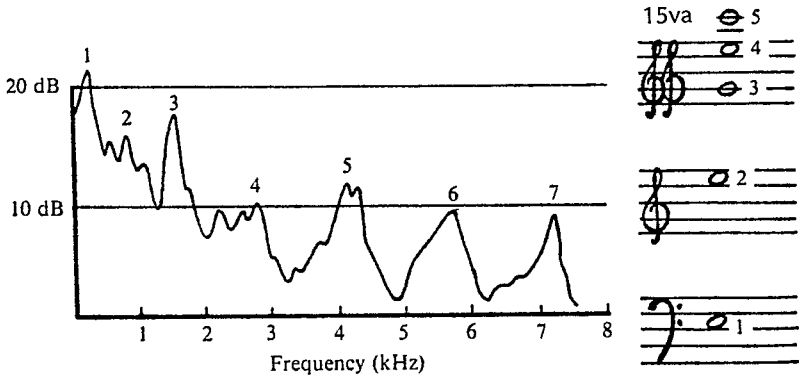


Fig. 3. Sound spectrum for E_3 marimba bar.[5]

It is not difficult to test the overtone tuning of a marimba by noting the position of the nodes as shown in figure 2. To suppress the fundamental mode and emphasize the first overtone, one should touch the bar firmly at the center and strike the bar either at one end or at a point about one-third of the way from the center to either end.

Resonators

Marimba resonators are cylindrical pipes tuned to the fundamental mode of the corresponding bars. A pipe with one closed end and one open end resonates when its acoustical length is one-fourth of a wavelength of the sound. The purpose of the tubular resonator is to emphasize the fundamental and also to increase the loudness, which is done at the expense of shortening the decay time of the sound. The statement is sometimes made that the resonator "prolongs" the sound, but this is incorrect. In a noisy environment or when played with other instruments in ensemble that impression may be conveyed, as can be understood by referring to figure 4.

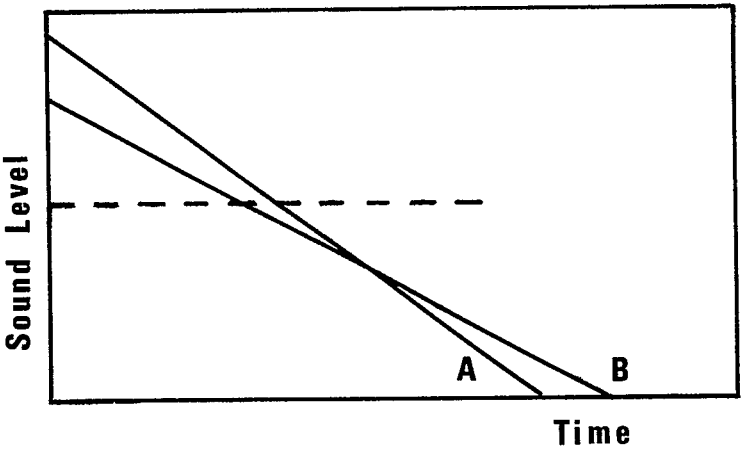


Fig. 4. Sound decay for a marimba bar with a resonator (curve A) and a bar without a resonator (curve B). If the background level (dashed line) is high enough, the sound represented by curve A may appear to be longer in duration.

Curve A, which represents the more rapid decay of a bar with a resonator, begins at a higher sound level than curve B, which represents the decay of a bar with no resonator. At some point in time the curves cross. If the level of background noise (dashed line in figure 4) is high enough, curve A may cross it after curve B, thus appearing to be longer in duration even though its decay time is shorter.

It is common practice to express the decay time of a sound as the time that would be required for the sound level to fall 60 dB (even though it fades into the background noise before then). We have measured the decay time (60 dB) of a rosewood marimba bar in the low register (E_3) to be 1.5 sec. with the resonator and 3.2 sec. without it. Decay times in the upper register are generally shorter; we measured 0.4 sec. and 0.5 sec. for an E_6 bar with and without the resonator, respectively. The corresponding decay times for synthetic bars are somewhat longer.

Tubes which are closed at one end and open at one end have resonances which are nearly the odd-numbered harmonics of the fundamental mode (i.e., the frequencies would be f, 3f, 5f, . . .). Thus a resonator tuned to the fundamental frequency of a marimba bar would not affect the first overtone, because the frequency of that mode is four times the fundamental frequency.

As an experiment, the author and percussionist Garry Kvistad designed a marimba with variable timbre which has a second set of resonators tuned to the first overtone of the bars. Each resonator is equipped with a vane that can partially or completely close the mouth of the tube; thus the timbre can be varied by adjusting the amount of closure.[6] This instrument was demonstrated at a meeting of the Acoustical Society of America held at Penn State University in 1977 (at this same meeting the Blackearth Group played a concert of percussion music for a large audience of acousticians).

The resonance frequency of an open-ended tube depends, to a small extent, on the environment near the end. This is because the reflection of standing sound waves within the tube does not take place precisely at the end of the tube walls. The tube acts as if it were a little longer than it really is; the difference between the effective length and the actual length is called the "end correction." The end correction for a cylindrical tube without a flange is 0.61 times its radius; if it has a large flange, the end correction is 0.85 times the radius.[7]

If the bar is close (within a distance of one radius, say) to the resonator tube, the resonance frequency will be affected by its presence. In particular, the resonance frequency will be lowered as the bar is moved closer to the tube. This effect can be used to compensate for changes in resonator frequency due to changes in ambient temperature. It works as follows: if the temperature rises, the speed of sound increases, and so does the resonance frequency of the tube. Moving the resonator closer to the bar, however, lowers the resonance frequency of the tube, so that it once again matches that of the bar. Some marimbas incorporate an adjustment of this nature, but I do not really know how much effect it has on the sound.

Xylophone

The *xylophone* is a close cousin to the marimba (or perhaps "uncle" is a better term, since the xylophone apparently has a longer history). Xylophones typically cover a range of 3 to 3½ octaves extending from F_4 or C_5 to C_8 (f = 349-4186 Hz) and may have bars of synthetic material or rosewood. Modern xylophones are nearly always equipped with tubular resonators to increase the loudness of the tone.

Xylophone bars are also cut with an arch on the underside, but the arch is not as deep as that of the marimba, since the first overtone is tuned to a musical 12th above the fundamental (that is, three times the frequency of the fundamental). Since a pipe closed at one end can also resonate at three times its fundamental resonant frequency, the 12th will also be reinforced by the resonator. This overtone boost, plus the hard mallets used to play it, give the xylophone a much crisper, brighter sound than the marimba. We have found that careful overtone tuning is usually ignored in the upper register, as in the case of the marimba.

The xylomarimba or xylorimba is a large xylophone with a 4½ to 5-octave range (C_3 or F_4 to C_8), and is occasionally used in solo work or in modern scores. Bass xylophones and keyboard xylophones have also been constructed.[8]

Xylophones of some type appear in most musical cultures, ancient and modern alike. Sometimes the instruments incorporate tuned gourd or bottle-type resonators. Sometimes the bars are mounted over a box-like cavity which appears to act more as a baffle than a resonator. A baffle amplifies the sound of a bar by reducing interference between sound radiated by the top and bottom surfaces of the bar as it vibrates.[9]

Vibraphone

A very popular bar percussion instrument is the *vibraphone* or *vibraharp,* as they are designated by different manufacturers. Vibes, as they are popularly called, usually consist of aluminum bars tuned over a three-octave range from F_3 to F_6 (f = 175-1397 Hz). The bars are deeply arched so that the first overtone has twice the frequency of the fundamental, as in the marimba. The aluminum bars tend to have a much longer decay time than the wood or synthetic bars of the marimba or xylophone, and so vibes are equipped with pedal-operated dampers.

The most distinctive feature of vibes, however, is the vibrato introduced by motor-driven discs at the top of the resonators, which alternately open and close the tubes. The vibrato produced by these rotating discs or pulsators consists of rather substantial fluctuation in amplitude ("intensity vibrato") and a barely detectable change in frequency ("pitch vibrato"). The speed of rotation of the discs may be adjusted to produce a slow vibe or a fast vibe. Often vibes are played without vibrato by switching off the motor. They are usually played with soft mallets or beaters, which produce a mellow tone, although some passages call for harder beaters.

Because vibraphone bars have a much longer decay time than do marimba and xylophone bars, the effect of the tubular resonators on decay time is more

dramatic. At 220 Hz (A_3), for example, we measure a decay time (60 dB) of 40 sec. without the resonator and 9 sec. with the tube full open. For A_5, we measure 24 sec. with the resonator closed and 8 sec. with it open. In the recording of sound level shown in figure 5, the intensity modulation, as well as the slow decay of the sound, can be clearly seen.

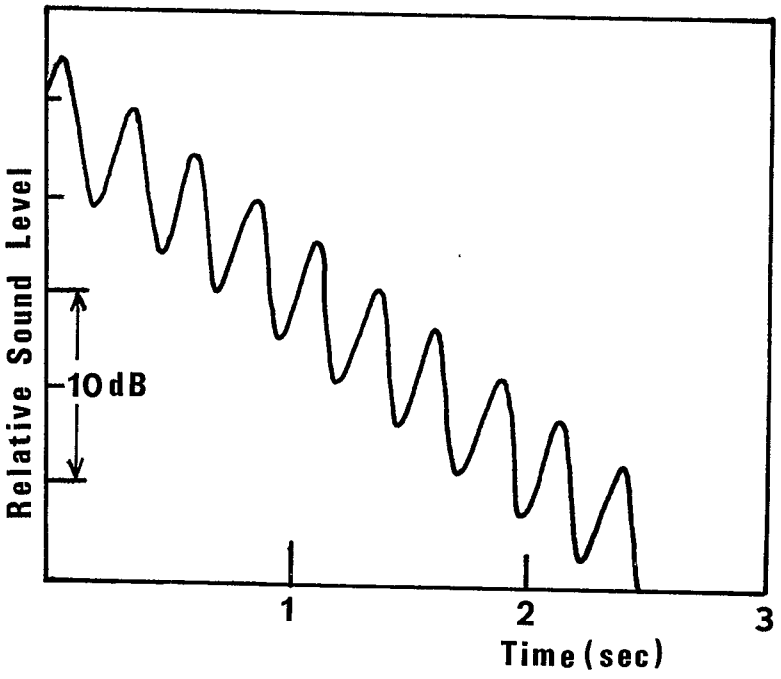


Fig. 5. Sound level recording for A_3 bar (f = 440 H.) with a vibe rate of about 4 H. The level fluctuates about 6 db., and the decay time (60 db.) is 7 sec.[10]

Triangles

Because of their many modes of vibration, triangles are characterized as having an indefinite pitch. They are normally steel rods bent into a triangle (usually, but not always, equilateral) with one open corner. Triangles are suspended by a cord from one of the closed corners, and are struck with a steel rod or hard beater.

Triangles are typically available in 15 cm, 20 cm, and 25 cm (6-, 8-, and 10-in.) sizes, although other sizes are also used. Sometimes one end of the rod is bent into a hook, or the ends may be turned down to smaller diameters than the rest of the triangle to alter the modes of vibration. The sound of the triangle depends on the strike point as well as the hardness of the beater. Single strokes are usually played on the base of the triangle and perpendicular to the plane. Normal strokes will tend to emphasize vibrations in the plane of the triangle. Figure 6 shows sound spectra for a 10-in. triangle at two different strike points with strokes parallel and perpendicular to the plane of the triangle.

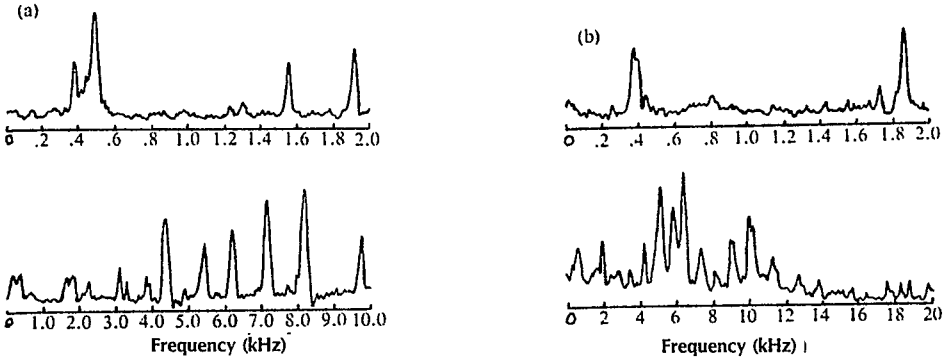


Fig. 6. Sound spectra for a 25-cm steel triangle: (a) struck in the plane and (b) struck perpendicular to the plane. Two frequency and amplitude ranges are shown in each case.[11]

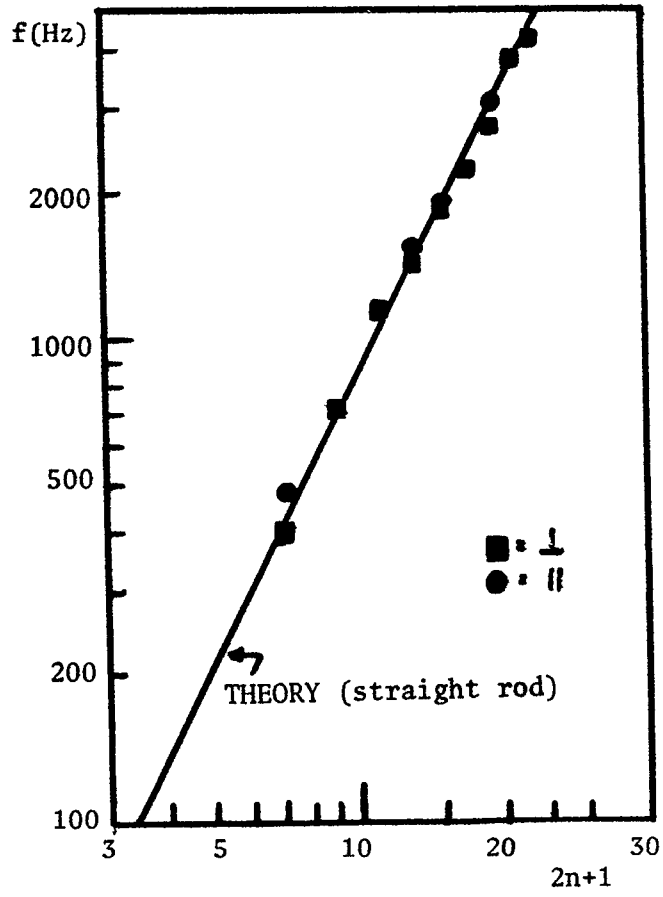


Fig. 7. Mode frequencies for a 25-cm steel triangle driven in the plane and perpendicular to the plane. The line gives the predicted frequencies for a steel rod of the same diameter and length.[12]

Since triangles are made of steel, single modes of vibration can be excited by driving a ferrite antenna from a radio by means of an audio amplifier to produce an oscillating magnetic field. At each resonance frequency, a small microphone can be used to locate the nodes and antinodes and thus identify the mode. Some of the modes of vibration of a 10-in. triangle are shown in figure 7, along with those calculated for a steel rod of the same length and diameter. The triangle modes show a surprisingly close correspondence to those of a straight rod.

Chimes

Chimes or *tubular bells* are an important type of percussion instrument with modes of vibration like those of a free bar. However, their timbre is like that of a bell.[13]

Concluding Remarks

In this paper we have discussed the modes of vibration of bars and their application to a number of bar percussion instruments. The author welcomes questions, critical comment, and feedback of any kind from readers, especially performing percussionists.

ENDNOTES

I wish to thank Garry Kvistad and Calvin Ross for their valuable contributions to these studies and Wallace Carroll for his generous support during the initial experiments.

[1]T. D. Rossing, *The Science of Sound* (Reading, Mass., Addison-Wesley, 1982), chap. 3

[2]The speed of sound in some materials is given below in table 2.

[3]Rossing, "Acoustics of Percussion Instruments," *Physics Teacher* 14 (1976):549.

[4]Ibid., p. 550.

[5]Ibid.

[6]G. Kvistad and T. D. Rossing, "Variable Timbre in Mallet Percussion Instruments," *Journal of the Acoustical Society of America* 61 (1977):21.

[7]L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics,* 2d ed. (New York: Wiley, 1962), p. 187 and Rossing, *Science of Sound,* p. 53.

[8]R. S. Brindle, *Contemporary Percussion* (London: Oxford University Press, 1970), pp. 35-45.

[9]Baffles are nearly always used to improve the efficiency of sound radiation from loudspeakers. Simple experiments that demonstrate the effect of a baffle are shown in Rossing, *Science of Sound,* fig. 20:18.

[10]Rossing, "Acoustics of Percussion Instruments," p. 552.

[11]Ibid., p. 554.

[12]Ibid., p. 555.

[13]Ibid. For a more detailed discussion of chimes and bells see p. 42 this issue.